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## 2 Fluid flow

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### 2.1 Introduction

From the macroscopic viewpoint of the engineer concerned with fluid transport, it is a convenient idealisation to treat fluid flow as that of a continuum, thereby neglecting the complex random motions at molecular level. Flow analysis is concerned with quantifying the flow variables throughout the flow field, as functions of time; these variables are velocity, pressure and density. This approach may be contrasted with that adopted in solid particle mechanics, where the focus of kinematic analysis is on the motions of individual particles.

Velocity is, of course, a vector, that is, it has magnitude and direction. In a 3-dimensional flow field, the components  $u$ ,  $v$  and  $w$  of the velocity vector  $\mathbf{V}$ , in the  $x$ ,  $y$  and  $z$  directions, respectively, can be written in functional form as follows:

$$\begin{aligned}u &= f_1(x, y, z, t) \\v &= f_2(x, y, z, t) \\z &= f_3(x, y, z, t)\end{aligned}$$

These components define the value of  $\mathbf{V}$  in space and time.

A **streamline** is defined as a continuous curve in the flow field that is everywhere tangential to the local velocity vector. It is thus a flow path.

### 2.2 Flow classification

Flow is described as steady at a particular location if the velocity vector at that location does not change with time; it is described as **unsteady** if the velocity vector changes with time. In mathematical terms these definitions are written as follows:

(1) steady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0;$$

(2) unsteady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

Flow is said to be **uniform** if the velocity vector is constant along the flow path or streamline. Conversely, flow is described as **non-uniform** if the velocity vector varies along the flow path. These definitions are expressed in mathematical terms as follows:

(1) uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t_0} = 0;$$

(2) non-uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t_0} \neq 0$$

The most regulated of the foregoing flow types is steady uniform flow such as occurs in pipes of fixed diameter having a constant discharge rate. An example of steady non-uniform flow is that which occurs upstream of a weir in a river having a steady discharge rate.

Examples of unsteady non-uniform flow include estuarine flows (due to variation in channel section and time-variation in flow associated with tides) and flood flows in rivers.

Flow is described as rotational if fluid elements undergo a rotation about their centres of mass and **irrotational** if no such rotation exists. Where there is a spatial velocity gradient in the flow field, as in very many real flow situations, e.g. in boundary layer flow, there is inevitably some degree of rotation. Flow is obviously rotational where the streamlines are curved.

Flow is described as compressible if the fluid undergoes a significant change in density along the flow path and **incompressible**, where there is no significant change in density. Flow of liquid is clearly incompressible flow. Flow of gases at the velocities normally encountered in sanitary engineering practice may also be regarded as incompressible. There may be significant density changes along the flow path in high velocity gas flows, however and hence thermodynamic behaviour must be taken into account in analysing such flow.

When fluid flow is confined by solid boundaries, such that random lateral mixing in a direction perpendicular to the flow is suppressed, flow is described as **laminar** i.e. flowing, as it were, in separate layers with minimal lateral momentum transfer between layers. Where there is significant lateral mixing and momentum transfer in a direction normal to the flow direction, flow is classified as **turbulent**. The criteria used to define laminar and turbulent flow conditions are discussed further in chapter 3.

### 2.3 Fluid acceleration

The acceleration at any point on a streamline can be expressed in terms of its tangential and normal components. The tangential component  $dv_s/dt$  may be derived as follows:

$$dv_s = \frac{\partial v_s}{\partial s} ds + \frac{\partial v_s}{\partial t} dt \quad (2.1a)$$

Hence

$$\frac{dv_s}{dt} = \frac{\partial v_s}{\partial s} \frac{ds}{dt} + \frac{\partial v_s}{\partial t} \quad (2.1b)$$

or

$$\frac{dv_s}{dt} = v_s \frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t} \quad (2.1c)$$

Equations (2.1) show the tangential acceleration to be the sum of the spatial or convective acceleration  $v_s(\partial v_s/\partial s)$  and the local or temporal acceleration  $\partial v_s/\partial t$ .

In steady flow  $\partial v_s/\partial t$  is zero and hence the steady flow acceleration is:

$$\frac{dv_s}{dt} = v_s \frac{\partial v_s}{\partial s} \quad (2.2)$$

The normal or centripetal acceleration  $dv_n/dt$  of a fluid element moving along a curved path or streamline can similarly be written as the sum of convective and temporal components:

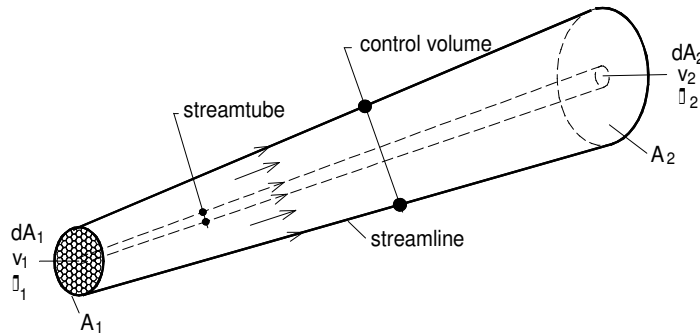
$$\frac{dv_n}{dt} = \frac{v_s^2}{R} + \frac{\partial v_n}{\partial t} \quad (2.3)$$

where R is the local radius of curvature of the streamline and  $v_n$  is the normal velocity.

### 2.4 Streamtube and control volume

The concepts of streamtube and control volume are widely used in fluid flow analysis. A streamtube is an elemental flow volume, the end areas of which are normal to the local flow directions and the peripheral surface of which is generated by streamlines. Flow into/out of the streamtube is through its end areas only; there is no flow normal to the peripheral surface since it is generated by streamlines and thus acts as a virtual solid boundary. The end areas are sufficiently small in extent that any variation in velocity over the cross-section may be neglected.

A bundle of adjacent streamtubes constitutes a control volume. A control volume has the same general characteristics as a streamtube except that there may be a variation in velocity over its end areas. These concepts are illustrated on Fig 2.1.



**Fig 2.1 Streamtube and control volume**

## 2.5 The continuity principle

The concepts of streamtube and control volume facilitate the application of the principle of mass conservation or the continuity principle, as it is known in fluid flow analysis. For example, under steady flow conditions, the mass of fluid contained within a streamtube or control volume does not change with time, hence the rate of mass flow out of such defined zones must equal its rate of inflow:

(1) streamtube

$$\rho_1 dA_1 v_1 = \rho_2 dA_2 v_2;$$

(2) control volume

$$\bar{\rho}_1 A_1 \bar{v}_1 = \bar{\rho}_2 A_2 \bar{v}_2;$$

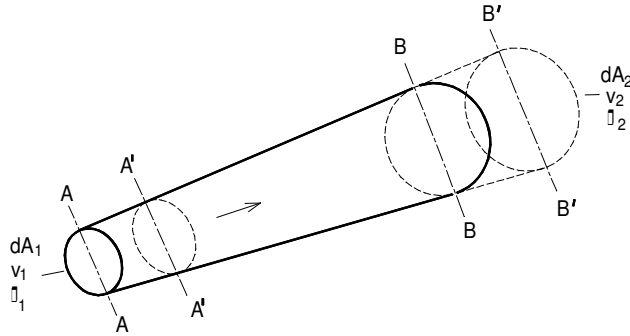
where  $\bar{\rho}$  and  $\bar{v}$  represent the averaged values of these parameters. The mass conservation principle will be applied repeatedly in later chapters to develop the appropriate form of the continuity equation for the problem on hand. In unsteady compressible flow for example, the fluid mass within the streamtube or control volume varies with time.

## 2.6 The momentum principle

Newton's second law relates force to the rate of change of momentum:

$$F = \frac{d}{dt}(mv)$$

Consider the application of this principle to steady flow through the streamtube illustrated on Fig 2.2:



**Fig 2.2 Streamtube flow**

At time zero the streamtube contains a mass of fluid between end areas AA and BB. In the following time interval dt this mass moves to the space between AA' and BB'.

$$\text{Initial momentum} = \sum_{BB}^{AA} dm v$$

$$\text{Final momentum} = \sum_{B'B'}^{A'A'} dm v$$

Since the flow is steady there is no change in momentum within the streamtube i.e. the momentum of the fluid in the space A'A'BB, which is common to both streamtubes, remains unaltered. Thus the change in momentum in the time dt can be written:

$$\text{Momentum change} = \sum_{BB}^{B'B'} dm v - \sum_{A'A'}^{AA} dm v$$

written in terms of  $\rho$ , dA and v, this becomes:

$$\text{Momentum change} = (\rho_2 dA_2 v_2 dt)v_2 - (\rho_1 dA_1 v_1 dt)v_1$$

The corresponding rate of change of momentum yields the magnitude of the applied force F:

$$F = \rho_2 dA_2 v_2^2 - \rho_1 dA_1 v_1^2 \quad (2.4)$$

The term  $\rho_2 dA_2 v_2^2$  represents the rate of outflow of momentum from the streamtube, while  $\rho_1 dA_1 v_1^2$  is its rate of inflow. Thus, the applied force corresponds to the difference in momentum flux across the streamtube end areas. It should be noted that this force is the net force applied to the fluid mass within the streamtube by the surrounding bulk fluid.

In unsteady flow, as discussed in chapters 7 and 10, the change in momentum of the fluid mass throughout the streamtube volume must also be taken into account.

Equation (2.4) may also be applied to a control volume:

$$F = \sum_{A_2} \rho_2 dA_2 v_2^2 - \sum_{A_1} \rho_1 dA_1 v_1^2$$

Written in terms of the mean velocity  $\bar{v}$ :

$$F = \beta_2 \rho_2 A_2 \bar{v}_2^2 - \beta_1 \rho_1 A_1 \bar{v}_1^2 \quad (2.5)$$

where  $\beta$  is the momentum correction factor (sometimes called the Boussinesq coefficient), which allows for the use of the mean velocity in the application of the momentum principle to control volumes. Its value is obtained as follows:

$$\beta \rho A \bar{v}^2 = \sum_A \rho dA v^2;$$

Hence

$$\beta = \frac{1}{A} \sum \left( \frac{v}{\bar{v}} \right)^2 dA \quad (2.6)$$

In turbulent flow,  $\beta$  is generally less than 1.1; in laminar flow,  $\beta$  is 1.33.

## 2.7 The energy principle

Consider the idealised flow of an elemental fluid mass along a streamline as depicted in Fig 2.3:

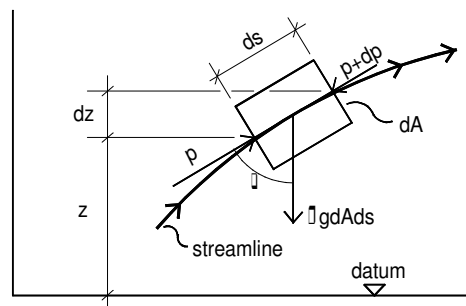
Applying Newton's second law to this elemental mass:

$$p dA - (p + dp) dA - \rho g dA ds \cos \theta = \rho dA ds \frac{dv}{dt} \quad (2.7)$$

Since  $ds \cos \theta = dz$  and  $dv/dt = v (dv/ds)$  in steady flow, equation (2.7) may be written as follows:

$$\frac{dp}{\rho} + g dz + v dv = 0 \quad (2.8)$$

This is the **Euler equation**; it relates to steady irrotational flow of a frictionless fluid along a streamline.



**Fig 2.3 Forces acting on an elemental fluid mass**

Integration of the Euler equation along a streamline yields:

$$\int \frac{dp}{\rho} + g z + \frac{v^2}{2} = \text{constant} \quad (2.9)$$

If the flow is incompressible i.e.  $\rho$  is constant and independent of  $p$ , eqn (2.9) becomes:

$$\frac{p}{\rho} + g z + \frac{v^2}{2} = \text{constant} \quad (2.10)$$

which is the **Bernoulli equation**; it relates to steady, irrotational, incompressible flow of a frictionless fluid along a streamline.

When related to liquid flows, the Bernoulli equation is usually written in the form:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad (2.11)$$

Each term in equation (2.11) has units of length (m) or "head". Their sum represents the total head relative to a datum defined by z. In dealing with incompressible flow the pressure term is conveniently taken as the gauge pressure.

When the flow is incompressible, integration of the pressure/density term in equation (2.9) requires reference to the equation of state for gases:

$$\frac{p}{\rho^\gamma} = \text{constant} \quad (1.13)$$

hence

$$\int \frac{dp}{\rho} = \frac{\gamma}{\gamma-1} \frac{p}{\rho}$$

or

$$\int \frac{dp}{\rho} = \frac{\gamma}{\gamma-1} R\Theta \quad (2.12)$$

which, from the definitions of R and g (refer section 1.5), can be written as follows:

$$\int \frac{dp}{\rho} = C_p \Theta \quad (2.13)$$

Thus, the integrated form of the Euler equation for steady compressible flow along a streamline becomes:

$$C_p \Theta + g z + \frac{v^2}{2} = \text{constant} \quad (2.14)$$

This is known as the **energy equation** and can also be written in the form:

$$C_v \Theta + \frac{P}{\rho} + g z + \frac{v^2}{2} = \text{constant} \quad (2.15)$$

since  $R = C_p - C_v$  and  $p/\rho = R\Theta$ , where P is the absolute pressure. Each term in the energy equations has units of energy per unit mass ( $\text{J kg}^{-1}$ ).

The foregoing Euler, Bernoulli and energy equations do not take into account the energy loss associated with the flow of all real fluids. Thus, in practice, the sum of the terms on the left-hand side of the Bernoulli and energy equations is not constant but decreases in the downstream direction along a streamline.

When dealing with practical flow situations, it is convenient to use the mean velocity  $\bar{v}$  and a kinetic energy factor  $\alpha$ , where:

$$\int \rho dA v v^2 = \alpha \rho A \bar{v} \bar{v}^2;$$

hence

$$\alpha = \frac{1}{A} \int \left( \frac{v}{\bar{v}} \right)^3 dA \quad (2.16)$$

The value of  $\alpha$  lies between 1.03 and 1.3 in turbulent flow and has the value of 2.0 in laminar flow.

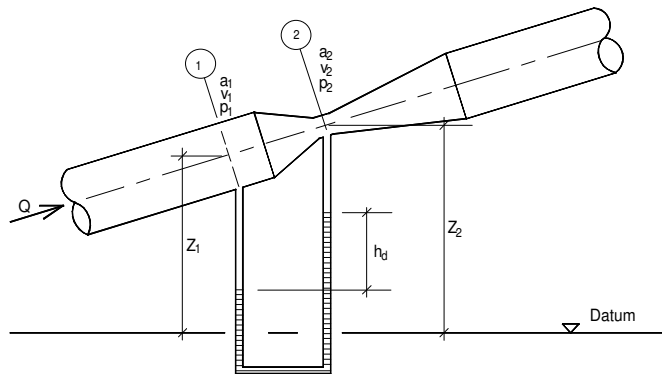
## 2.8 Applications of continuity, energy and momentum principles

### 2.8.1 Incompressible flow

Many examples of the application of the continuity, energy and momentum principles are presented in later sections of this book. In this section, the illustrative examples are confined to steady flow problems.

The discharge rate through differential head flow devices such as the Venturi meter and orifice plate meters is derived from application of the Bernoulli and continuity equations. Assuming incompressible flow and neglecting losses, the Bernoulli equation may be applied to the central streamline flow of the Venturi meter illustrated in Fig 2.4. :

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 \quad (2.17)$$



**Fig 2.4** Venturi meter

Similarly, the continuity equation may be applied to the flow through sections 1 and 2:

$$Q = a_1 v_1 = a_2 v_2 \quad (2.18)$$

Combining equations (2.17) and (2.18), the discharge  $Q$  can be expressed as a function of the upstream and throat cross-sections and the differential head:

$$Q = a_2 \left[ \left( \frac{2g}{1 - (a_2/a_1)^2} \right) \left( \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right) \right]^{0.5} \quad (2.19)$$

Also

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = h_d \left( \frac{\rho_m}{\rho} - 1 \right) \quad (2.20)$$

where  $h_d$  is the differential head of manometer fluid,  $\rho_m$  is the manometer fluid density. Because of flow losses between the upstream and throat pressure tapings, the measured differential head will exceed the theoretical value. This is taken into account in practical applications by introducing a discharge coefficient  $C_d$ . The practical discharge equation thus becomes:

$$Q = C_d a_2 \left[ \frac{2gh_d \left( \frac{\rho_m}{\rho} - 1 \right)}{1 - (a_2 / a_1)^2} \right]^{0.5} \quad (2.21)$$

The value of  $C_d$  lies between 0.96 and 0.99 for Venturi meters and in the range 0.60 to 0.63 for orifice plates. The diameter ratio  $d_2/d_1$  is typically in the range 0.3 to 0.7.

The Pitot tube, illustrated in (Fig 2.5), is a flow-measuring device which senses the kinetic streamline head at a point and hence can be used for flow velocity traversing. Writing the Bernoulli terms for the upstream section (1) and the stagnation point section (2):

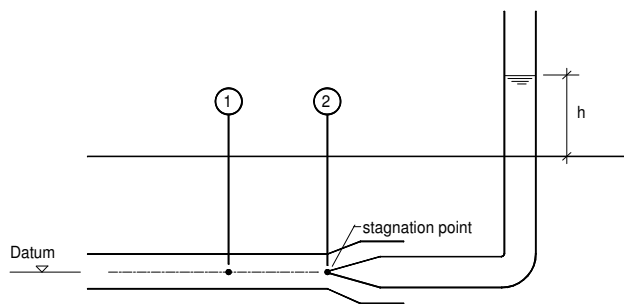
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + 0$$

Hence

$$v_1 = \sqrt{2g \left( \frac{p_2 - p_1}{\rho g} \right)} \quad (2.22)$$

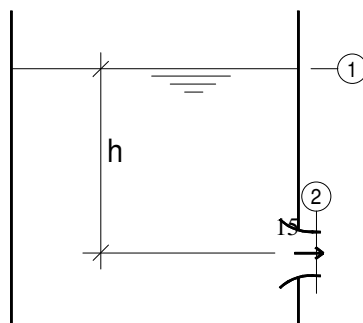
or

$$v_1 = \sqrt{2gh} \quad (2.23)$$



**Fig 2.5 Pitot tube**

The velocity given by the foregoing theoretical derivations is slightly too large and requires to be modified by a correction factor  $C_v$  which has a value typically in the range 0.95 to 1.0. The Bernoulli and continuity equations can also be applied to evaluate the velocity of discharge through a submerged orifice, as in Fig 2.6.



**Fig 2.6 Submerged orifice**

(1) Bernoulli

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + 0 \quad (2.24)$$

continuity

$$a_1 v_1 = a_2 v_2 \quad (2.25)$$

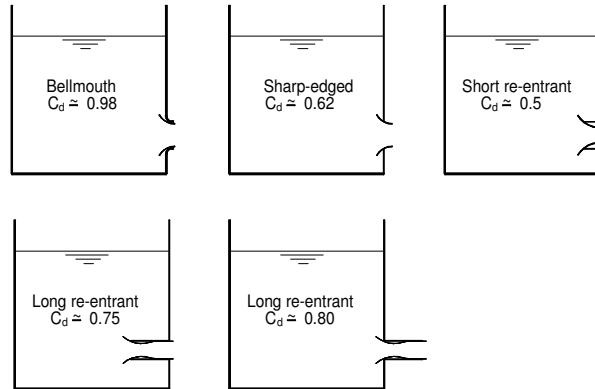
Combining equations (2.25) and (2.26) and assuming that  $p_1$  and  $p_2$  are both equal to atmospheric pressure:

$$v_2 = \left[ \frac{2gh}{1 - (a_2 / a_1)^2} \right]^{0.5} \quad (2.26)$$

The actual velocity is found to be somewhat less than the theoretical value. The cross-sectional area of the discharge jet may be effectively equal to the orifice area for a bellmouth orifice, reducing to about 0.6 of the orifice area for a sharp-edged orifice. Assuming that the velocity of approach is negligible, the discharge through a submerged orifice may be written in practical form as follows:

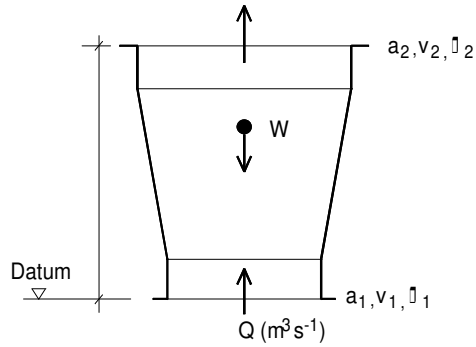
$$Q_o = C_d a_o \sqrt{2gh} \quad (2.27)$$

where  $Q_o$  is the orifice discharge,  $a_o$  is the orifice area and  $C_d$  is a discharge coefficient, the value of which generally lies in the range 0.50 to 0.98, depending on orifice geometry, as shown in Fig 2.7 (Featherstone & Nalluri 1988).



**Fig 2.7 Orifice discharge coefficients**

Computation of the forces exerted on pipe fittings such as the taper, illustrated on Fig 2.8 or the bend, illustrated in Fig 2.9, requires simultaneous application of the continuity, energy and momentum principles.



**Fig 2.8 Vertical pipe taper**

Applied to the vertical taper:

continuity

$$Q = v_1 a_1 = v_2 a_2 \quad (2.28)$$

energy

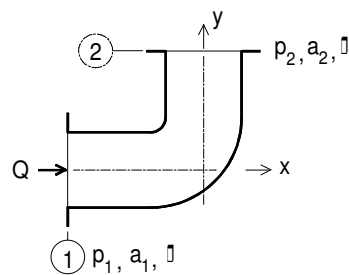
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + 0 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h \quad (2.29)$$

momentum

$$p_1 a_1 - p_2 a_2 - W + F_T = \rho Q (v_2 - v_1) \quad (2.30)$$

where  $F_T$  is the force exerted on the water mass in the taper (positive upward) and  $W$  is the weight of water within the taper. Thus, the force applied to the taper, which is the unknown quantity generally sought by the designer, is  $-F_T$ . When  $Q$  and the dimensions of the taper are specified, the magnitude of  $F_T$  can be computed from the foregoing continuity, energy and momentum equations.

The force exerted by steady flow through a horizontal pipe bend, as illustrated on Fig 2.9, is also found from application of the continuity, energy and momentum equations:



**Fig 2.9 Horizontal pipe bend**

continuity

$$Q = v_1 a_1 = v_2 a_2 \quad (2.31)$$

energy

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad (2.32)$$

(1) momentum, x-direction

$$p_1 a_1 + F_x = -\rho Q v_1 \quad (2.33)$$

momentum, y-direction

$$-p_2 a_2 + F_y = \rho Q v_2 \quad (2.34)$$

where  $F_x$  and  $F_y$  are the forces exerted on the fluid in the x and y directions, respectively, and  $-F_x$  and  $-F_y$  are the corresponding forces exerted by the flow on the bend.

### 2.8.2 Compressible flow

Compressible flow is marked by variations in fluid density along the flow path. The circumstances in which the related thermodynamic consequences must be taken into account can be examined as follows. The maximum temperature rise along a streamline occurs when the velocity is reduced to zero, that is, at the stagnation point. Applying the energy eqn (2.14) to this flow situation:

$$C_p \Theta + \frac{v^2}{2} = C_p (\Theta - \Delta\Theta)$$

upstream section      stagnation point

where  $\Delta\Theta$  is the temperature rise. This relationship can be written in the following form:

$$\frac{\Delta\Theta}{\Theta} = \frac{v^2}{2C_p \Theta} \quad (2.35)$$

For practical design purposes it may be assumed that, where the potential incremental temperature change along the flow path is less than 1 per cent, flow may be regarded as incompressible. Applying this criterion to eqn (2.35), the limiting value of  $v$  is found to be:

$$v = \sqrt{0.02 C_p \Theta} \quad (2.36)$$

giving a limit value for air at 10 °C of 75.4 ms<sup>-1</sup>.

Where, however, thermodynamic changes are significant the flow must be treated as compressible flow. Consider, for example, the steady discharge of a gas from a pipe or reservoir through an orifice or nozzle, as illustrated in Fig 2.10. The flow is defined by the following correlations:

(1) continuity

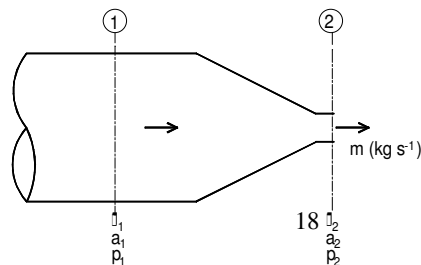
$$m = \rho_1 a_1 v_1 = \rho_2 a_2 v_2 \quad (2.37)$$

(2) energy

$$\frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1} + \frac{v_1^2}{2} = \frac{\gamma}{\gamma-1} \frac{p_2}{\rho_2} + \frac{v_2^2}{2} \quad (2.38)$$

(3) pressure/density

$$\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma} \quad (2.39)$$



**Fig 2.10 Gas discharge through an orifice**

These correlations assume idealized flow with zero energy loss. Using these equations, the mass discharge rate  $m$  ( $\text{kg s}^{-1}$ ) can be expressed in terms of known parameter values,  $P_1$ ,  $\rho_1$ , and  $P_2$ , and the sectional areas  $a_1$  and  $a_2$ :

$$m = a_2 \left[ \left( \frac{P_1 \rho_1}{(P_2 / P_1)^{-2/\gamma} - (a_2 / a_1)^2} \right) \left( \frac{2\gamma}{\gamma-1} \right) \left( 1 - (P_2 / P_1)^{(\gamma-1)/\gamma} \right) \right]^{0.5} \quad (2.40)$$

The foregoing expression is valid for subsonic flow conditions, that is, up to the pressure ratio at which the velocity at the orifice or nozzle throat reaches the sonic value. The sonic velocity  $\alpha$  is the velocity of transmission of a weak pressure wave through the gas. Its magnitude (Benedict 1983) is

$$\alpha = \sqrt{\gamma P_2 / \rho_2} \quad (2.41)$$

When the critical pressure ratio is exceeded, flow control changes from a dependency on the pressure ratio  $P_2/P_1$  to a dependency on the upstream pressure  $P_1$ , the upstream flow remains subsonic, and the velocity through the throat remains at sonic value. The critical pressure ratio may be found by applying the energy equation to the upstream and throat sections at the onset of sonic flow at the throat section:

$$\left( \frac{\gamma}{\gamma-1} \right) \frac{P_1}{\rho_1} + \frac{v_1^2}{2} = \left( \frac{\gamma}{\gamma-1} \right) \frac{P_2}{\rho_2} + \frac{\alpha^2}{2} \quad (2.42)$$

Combining eqns (2.41), (2.42) and (2.39) and neglecting  $v_1$  (this is equivalent to assuming  $p_1$  as the upstream stagnation pressure), the **critical pressure ratio** is found to be:

$$\frac{P_2}{P_1} = \left( \frac{2}{\gamma+1} \right)^{\gamma/(\gamma-1)} \quad (2.43)$$

The critical pressure ratio for air ( $\gamma = 1.4$ ) is found from eqn (2.43) to be 0.528.

When the pressure ratio is less than the above limiting value, the rate of discharge through the orifice or nozzle is found by inserting the limiting value of  $P_2/P_1$  in the discharge eqn (2.40). By omitting the area ratio term,  $a_2/a_1$ , the resulting discharge expression is found to be:

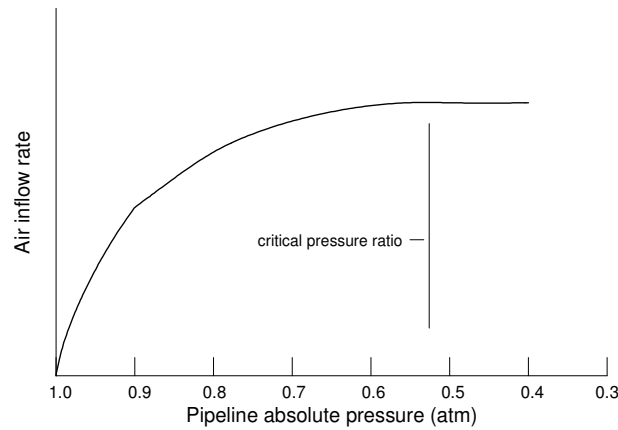
$$m = a_2 \left[ P_1 \rho_1 \gamma \left( \frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)} \right]^{0.5} \quad (2.44)$$

Equations (2.40) and (2.44) are theoretical expressions for the mass discharge rate of a compressible fluid through an orifice or nozzle. In practical computation these expressions are modified by the introduction of a discharge coefficient which allows for variations from ideal behaviour and in particular for flow contraction effects associated with the geometry of the orifice or nozzle.

The admission of air into a pipeline through an air valve consequent on a drop in pipeline pressure below atmospheric pressure provides a practical example of the foregoing compressible flow behaviour from the water engineering field, as illustrated in Fig 2.11. In this instance the external pressure ( $P_1$ ) remains constant at atmospheric pressure. The inflow of air through the valve orifice increases with the decrease in the internal pipeline pressure ( $P_2$ ) in accordance with flow eqn (2.40) until the critical pressure ratio (eqn (2.43)) is reached. Any further drop in internal pressure does not cause a corresponding increase in the rate of inflow since the latter now depends only on  $P_1$ , that is, atmospheric pressure, which is constant. An orifice operating under such conditions is sometimes described, for obvious reasons, as being “choked”.

## 2.9 Resistance to fluid flow

The resistance to fluid flow arises primarily from the drag forces exerted on flowing fluids by the solid boundary surfaces of flow conduits. This drag results from the fact that there is zero slippage or relative movement at the contact interface between a flowing fluid and a solid surface, resulting in high shear rates in the adjacent boundary



**Fig 2.11** Air entry rate to a pipeline through an air valve

fluid layer. This shear deformation is manifested as a spatial velocity gradient in a direction normal to the boundary surface, decreasing in magnitude with distance from the boundary.

The existence of a velocity gradient implies a causative shear stress, which is essential to maintain flow and which is a measure of the resistance to flow. Where the flow is laminar, that is, where there is no turbulence in the flow, the local shear stress/velocity gradient ratio is a constant. This constant is by definition the fluid viscosity  $\mu$ .

Where, however, flow conditions are turbulent, as is generally the case in civil engineering hydraulics, the correlation of shear stress and velocity gradient becomes more complex, being a flow property rather than a fluid property. The nature of turbulent flow and flow resistance in turbulent boundary layers is discussed in Chapter 3.

## References

Benedict, R. P. (1983) Fundamentals of gas Dynamics. John Wiley, New York.  
Featherstone, R. E. and Nalluri, C. (1988). Civil Engineering Hydraulics, (2<sup>nd</sup> edn).

### Related reading

Francis, J. R. D. and Minton, P. (1984) Civil Engineering Hydraulics, (5<sup>th</sup> edn), Edward Arnold, London.  
Shames, I. H. (1983) Mechanics of Fluids, (2<sup>nd</sup> edn), McGraw Hill, New York.  
Streeter, V. L. (1966) Fluid Mechanics (4<sup>th</sup> edn), McGraw Hill, New York.